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Series: “Modern Mathematics for Engineers”

Tamara G. Stryzhak

Minimax criteria of stability

$$L = T - \Pi$$

$$\max_{q} \left\langle \min_{\dot{q}} L\left(t, \dot{q}_j, q_j\right) \right\rangle$$

Tamara G. Stryzhak

THE MINIMAX CRITERION FOR STABILITY

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Dedicated

to the 110th anniversary
of the foundation
of the Kiev Polytechnic
Institute,
and to those students
of engineering
whom I taught to
mathematics
during the last 40 years

With gratitudes,
prof. Tamara Stryzhak

Series: “Modern Mathematics for Engineers”

Lectures for the trainee-students of IAESTE

Educational publication

Lectures
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∰ **Series: "Modern Mathematics for Engineers"**

Minimax criteria of stability

Professor NTUU "KPI"
Tamara Stryzhak

Lectures

This publication presents the “Minimax criterion of stability”. The “Minimax criterion of stability” is a criterion sufficient for determining the motion stability of a mechanical system under the influence of small-amplitude high-frequency parameter oscillations. This criterion allows the calculation of stability conditions of the oscillations of a non-stable mechanical system. It does not involve the use of motion equations, rather, it uses only the Langrangian function L= T – Π.

All remarks and recommendations are greatly appreciated and will be taken into considerations in subsequent editions.

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Introduction

(a) Acknowledgements

> "Mathematical Truth stays put in centuries,
> Metaphysical Ghosts disappear
> like delirium of the sick."
> *Voltaire*

Minimax Criterion of Stability is the first volume within the project *Modern Mathematics for Engineers*.

The project *Modern Mathematics for Engineers* was launched by AUS–DAAD and the National Committee IAESTE Ukraine. The purpose of the project is to publish the most essential mathematical results, to present them with enough clarity for engineers to apply and, ante omnia, to realize the scientific exchange in the sphere of applied mathematics, the oscillation theory, theoretical mechanics, etc.

We expect to realize this purpose first of all with the help of IAESTE trainee students.

The first trainee students who took part in the project:

- David Ellis (Imperial College of London, Great Britain);
- Andrea Trautsamwieser (Technical University of Vienna, Austria);
- Sarah Burden (University of St. Andrews, UK);
- Leonard Neuhaus (Ludwig Maximilians University of Munich, Germany);
- Erkan Koc (University of Bonn, Germany);
- Henning Hans Petzka (University of Aahen, Germany);
- Eunice Kim (Carnegie Mellon University, USA);
- Peter Smith (Queen's University Belfast, UK);
- Heide Gieber (Vienna University of Technology, Austria);
- Laura Lambertz (RWTH Aachen, Germany);

- Mirjana Vukelja (ETH Zurich, Switzerland);
- Bettina Carina Sieber (TU Munich, Germany).

We invite for cooperation all mathematicians and engineers who are interested in having their scientific works published in the frame of the project.

We were inspired to launch the project by the positive experience of the researchers of the Californian University, who published the monograph *Modern Mathematics for Engineers*[1] more than half a century ago, in 1956. This work was very successful. As a matter of fact this monograph laid the solid foundation for the applied mathematics to develop successfully and steadily. This monograph is an excellent example to follow. Thus, like 50 years ago, we are launching the project with researches dedicated to the pendulum. We can hardly remember any other mechanism which is simpler than the pendulum, the mechanism whose scientific life has been so rich in application in different spheres of organic and inorganic nature, as the pendulum has lived a long and rich in discoveries life. I think it is the right time to build a monument to a simple and meaningful device.

Our contribution into realization of this project included taking the following steps:

1) publishing the monograph *Research Methods of the Pendulum Dynamic Systems* [4];
2) receiving “Minimax Criterion of Stability” and applying it in order to research a number of mathematical models and proved, in particular, that any position of the pendulum, even a horizontal one in the vertical plane, can be made stable with the help of the suspension point oscillations;
3) building an installation to demonstrate our theoretical results.

All in all we tried our best to state the results avoiding proving theorems as their translation from Russian into English might have some inadequacy as grammars of these languages have a certain level of ambiguity. Instead of proving the theorems we shall follow Lopital’s words, the author of the first textbook on Mathematical Analysis, and say, “We pass the word of honor that the theorem is true”.

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1 Edited by Edwin F. Beckenbach (Professor of Mathematics, University of California, Los Angeles). New York, Toronto, London: McGraw-Hill 1956.

(b) Oscillations – what are they?

> "Nature prefers oscillating motions in all demonstrations of life.
> Not without reason we can assume that there is some
> optimality property at the back of this phenomenon."[8]

It would be rather difficult to mention all publications about the Pendulum. As a matter of fact, oscillations of pendulum systems were researched by Galileo, Newton, Euler, Huygens and many other scientists who made a great contribution into studying the mechanism of the pendulum motion, which was of great importance for the history of mankind's discoveries and technical progress. The pendulum has been used in clocks to define time, in special devices to measure the terrestrial gravitation, as a plumb line in building to define the vertical, etc. The Earth's rotation was proved with the help of the pendulum oscillations as well.

Pendulum systems are, as a rule, non-linear and require specific methods of research. For instance, creation of the elliptical function theory by Abel, Jacoby, and Weierstrass is also connected with the research of the mathematical pendulum oscillations.

Nowadays due to the mathematicization of research in different sciences there has been an increasing interest in studying motions of pendulum systems. The following special terı7 s have appeared: the pendulum law of the population migration, the pendulum law of the rhythm regulator action, the pendulum of emotions, etc.

It is a well-known fact that all living organisms have so called biological clocks, at the basis of which there is an oscillating system – a non-linear oscillator. It is well known that the vestibular apparatus of animals and humans contains three non-linear pendulum systems, which are located in three mutually perpendicular planes. Oscillations are present everywhere: in the opening of a flower at the sunrise, in the growth of an embryo, in germinating of a grain, in the heart beating, in the work of a dental drill and a jackhammer, in the rise and fall of the tide. Besides, wherever there is life, there are oscillations at the cellular level.

The nature of forces which cause oscillations is variable, but the result of these forces is the same, namely - oscillations.

This information bears a descriptive character as we would like to attract the readers' attention to the oscillation theory to convince them of the fact that oscillation processes imply something much deeper than what we actually know: that the source of the oscillating processes is a hidden potential force and kinetic energy. Oscillations of a non-linear oscillator can be forced or can have a free-running character (such as the heart or aorta beating, biorhythms, etc.). In fact, the pendulum motion laws, periodic or almost-periodic oscillations, are immanent in the whole physical world.

A human being receives the main information about the outer world via sound and light oscillations, analyzing which scientists use pendulum systems. The latter are found in different engineering tasks. For example, oscillations in electrical and mechanical systems, rocking of ships on water, oscillations of satellites, vibration of the hull and the wings of planes, movements of cables, chains, travelling or gantry cranes, etc. All these phenomena are explained with analogous differential equations which describe motions.

At this point our introduction is completed and we turn to the beautiful algorithmic mathematical language: *“A” is given, “B” is to be proved.*